Structured perturbations - normwise and componentwise

Abstract: Using structure in matrix algorithms sometimes leads to a dramatic improvement. For example, solving a linear system with Toeplitz matrix can be solved in O(n^2) time, the same time as to print the matrix inverse. Such a solver only permits Toeplitz perturbations, thus a stability analysis should not use general perturbations.

Explicit formulas are given for structured condition numbers, using normwise and componentwise perturbations. One result is that for specific right hand side the Toeplitz condition number can be exponentially better than for general perturbations.

For matrix inversion things change completely. For commonly used structures the general and the structured condition number coincide. In other words, amongst the worst perturbations is a structured one. The same is true for the distance to the nearest singular matrix. It follows that the Eckart-Young Theorem is valid for structured perturbations.

For componentwise perturbations things change again completely. For commonly used structures there are examples with structured condition number O(1), but arbitrarily large general condition number.