Perron-Frobenius Theory for complex matrices

Abstract: It is not difficult to see that the Eckart-Young Theorem does not carry over to componentwise perturbations. One "reason" is that the componentwise condition number of an nxn matrix can be computed in O(n^3) operations, whereas computation of the componentwise distance to the nearest singular matrix is NP-hard.

One may ask whether there is a relation between the reciprocal of the condition number and the distance to the nearest singular matrix for componentwise perturbations. The answer is in the affirmative, and the solution of this problem leads to a generalization of Perron-Frobenius Theory from non-negative to general real and complex matrices. A number of results nicely carry over.

However, there a number of open problems. They are based on the theory but can be formulated in easy matrix terms.

One example is the following. Given a real (complex) nxn matrix A such that |A|e=ne, where e is the vector of 1's and the absolute value is taken componentwise. Show existence of a non-trivial real (complex) vector x with |Ax| >= |x|. Here comparison is componentwise as well, i.e. u >= v iff u_i >= v_i for all i.