Title: Optimising the Kemeny Constant for a Markov Chain

Abstract: Suppose that A is an irreducible stochastic matrix of order n. Then A can be thought of as the transition matrix for a Markov chain, and various quantities associated with A give insight into the nature of the corresponding Markov chain. For example, it is well–known that if A is primitive, then the stationary distribution vector w for A – that is, the left Perron vector of A normalised so that its entries sum to 1 – describes the limiting behaviour of the Markov chain, since the iterates of the latter converge to w independently of the initial distribution.

Somewhat less well–known is the Kemeny constant $K(A)$ for the Markov chain associated with A. Denoting the eigenvalues of A by $1, \lambda_2, \ldots, \lambda_n$, $K(A)$ is given by $K(A) = \sum_{j=2}^{n} \frac{1}{1-\lambda_j}$. A remarkable result of Kemeny asserts that for each $i = 1, \ldots, n, \sum_{j=1}^{n} m_{i,j} w_j = K(A) + 1$, where for $i, j = 1, \ldots, n, m_{i,j}$ denotes the mean first passage time from state i to state j. Thus, the Kemeny constant can be interpreted in terms of the expected number of time steps taken to arrive at a randomly chosen state, starting from initial state i. In particular, if $K(A)$ is small, then we can think of the Markov chain corresponding to A as possessing good mixing properties.

This last observation motivates our interest in identifying stochastic matrices for which the corresponding Kemeny constant is as small as possible. In this talk, we will give a short overview of the Kemeny constant, and discuss some results dealing with the problem of minimising the Kemeny constant over stochastic matrices that are subject various constraints. In particular, we will find the minimum value of the Kemeny constant for stochastic matrices having a specified stationary distribution vector, and characterise those stochastic matrices yielding that minimum value.