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Part VI Abstracts - Contributed Talks

INFORMATION

Continuing the tradition of conferences dedicated to days full of matrices, in 2013 Faculty of Science, Department of Mathematics and Informatics, Faculty of Mathematics and Computer Sciecnce of Adam Mickiewicz University and ALA Group will organize MAT-TRIAD 2013 in Herceg Novi, Montenegro.

Since matrix theory is used in all parts of pure and applied mathematics, as well as in other sciences, industry and technology, the current increasing pace of scientific and technological development motivates more and more researchers all over the world to investigate the problems and construct new models and approaches in this field.

Similar to its predecessors, this MAT-TRIAD conference aims to bring together researchers that share interests in a variety of aspects of matrix analysis and its applications, and to offer them a possibility to discuss state of the art developments in their fields.

A special issue of CEJM (Central European Journal of Mathematics) related to matrix problems, with an essential part based on the best papers of participants will be published after the conference. All the details about paper submission will be announced later on.

The programme will cover different aspects of applied and numerical linear algebra, with the emphasis on: - recent developments in matrix theory in general - connections between matrix and graph theory - applications of linear algebra tools in statistics - matrix models in industry and sciences

Scientific programme of this meeting will include plenary talks, sessions with contributed talks, and two short courses delivered by experienced lecturers for graduate students as well as other conference participants.

The work of young scientists has still a special position in the MAT TRIAD 2013. The best talk of graduate students or scientists with a recently completed Ph.D. will be awarded. Prize-winning works will be widely publicized and promoted by the conference.

Organizers:

- Faculty of Science, University of Novi Sad
- Faculty of Mathematics and Computer Science of Adam Mickiewicz University
- ALA Group

Organizing comittee:

- Ljiljana Cvetković (Serbia) - chair
- Francisco Carvalho (Portugal)
- Ksenija Doroslovački (Serbia)
- Vladimir Kostić (Serbia)

Scientific comittee:

- Tomasz Szulc (Poland) - chair
- Natália Bebiano (Portugal)
- Ljiljana Cvetković (Serbia)
- Heike Faßbender (Germany)
- Simo Puntanen (Finland)

Invited speakers:

- Richard A. Brualdi (USA)
  Title: *Alternating Sign Matrices: History, Patterns, Completions, and Spectral Radius*

- Stephen J. Kirkland (Ireland)
  Title: *Optimising the Kemeny Constant for a Markov Chain*

Invited lecturers:

- Siegfried M. Rump (Germany)
  Lecture 1: *Structured perturbations - normwise and componentwise*
  Lecture 2: *Perron-Frobenius Theory for complex matrices*

- Adi Ben-Israel (USA)
  Lecture 1: *The Moore–Penrose inverse*
  Lecture 2: *Selected applications*

Winners of YSA 2011:

- Olivia Walch (USA)

- Paulo Canas Rodrigues (Portugal)
  Title: *The role of matrix analysis in statistical genetics*

PROGRAM

Monday, September 16

8:45- 8:55 OPENING

9:00- 9:45
Invited speaker: Richard A. Brualdi
Title: *Alternating Sign Matrices: History, Patterns, Completions, and Spectral Radius*
Chairperson: Tomasz Szulc

9:45 - 10:00 BREAK

10:00 - 11:30
Invited lecturer: Siegfried M. Rump
Lecture 1: *Perron-Frobenius Theory for complex matrices*
Chairperson: Vladimir Kostić

11:30 - 11:45 BREAK

Short presentations

Chairperson: Natalia Bebiano

11:45 - 12:10 Tomasz Szulc : *Estimation of extremal singular values*

12:10 - 12:35 J.A. Armario : *Determinants of* $(-1, 1)$*-matrices of the skew-symmetric type: a cocyclic approach*

12:35 - 13:00 D. Wojtera-Tyrakowska : *Extremal values of modified equiradial and equimodular sets of a given matrix*

13:00 - 15:00 LUNCH BREAK

Chairperson: Miroslav Rozložnik

15:00 - 15:25 Rafael Bru : *Jordan structure of rank one updated matrix*

15:25 - 15:50 Ana Nata : *The indefinite numerical range of banded biperiodic Toeplitz operators*

15:50 - 16:15 Natália Bebiano : *Revisiting the inverse field of values problem*

16:15 - 16:40 Apostolos Hadjidimos : *On Brauer-Ostrowski and Brualdi sets*

16:40 - 17:00 BREAK

Chairperson: Rafael Bru

17:00 - 17:25 Ernest Šanca : *Diagonal dominance in Euclidian norm*

17:25 - 17:50 Sanja Konjik : *Symplectic linear algebra over Colombeau-generalized numbers*

17:50 - 18:15 Peter Junghanns : *Structured matrices in the numerical analysis of singular integral equations*

18:15 - 18:40 Yücel Çenesiz : *Using the fundamental matrix properties in collocation methods*

Tuesday, September 17

9:00- 9:45
Invited speaker: Paulo Canas Rodrigues
Title : *The role of matrix analysis in statistical genetics*
Chairperson: Heike Fassbender

9:45 - 10:00 BREAK

10:00 - 11:30
Invited lecturer : Adi Ben-Israel
Lecture 1: *The Moore–Penrose inverse*
Chairperson: Vladimir Kostić

11:30 - 11:45 BREAK

Short presentations

Chairperson: Augustyn Markiewicz

11:45 - 12:10 Simo Puntanen : *All about the* $\perp$

12:10 - 12:35 Francisco Carvalho : *Orthogonal and error orthogonal models and perfect families*

12:35 - 13:00 Anuradha Roy : *Supervised classifiers of ultra high-dimensional higher-order data with locally doubly exchangeable covariance structure*

13:00 - 15:00 LUNCH BREAK

Chairperson: Francisco Carvalho

15:00 - 15:25 Miguel Fonseca : *Non-normal linear mixed models*

15:25 - 15:50 Ricardo Covas : *Models with homoscedastic otyhogonal partition, BQUE and mixed models*

15:50 - 16:15 Augustyn Markiewicz : *On some matrix aspects of design optimality in interference model*

16:15 - 16:40 Katarzyna Filipiak : *Optimal designs under the generalized extended growth curve model*

16:40 - 17:00 BREAK

Chairperson: Katarzyna Filipiak

17:00 - 17:25 Wojciech Tadej : *Affine Hadamard families stemming from Kronecker products of Fourier matrices*

17:25 - 17:50 Jakub Kierzkowski : *On convergence of SOR-like methods for solving the Sylvester equation*

17:50 - 18:15 Rute Lemos : *Reverse Heinz-Kato-Furuta inequality*

18:15 - 18:40 Karla Rost : *Inversion of Bezoutians*

Wednesday, September 18

9:00- 9:45
Invited speaker: Stephen Kirkland
Title: *Optimizing the Kemeny constant for a Markov chain*
Chairperson: Simo Puntanen

9:45 - 10:00 BREAK

10:00 - 11:30
Invited lecturer: Siegfried M. Rump
Lecture 2: *Structured perturbations-normwise and componentwise*
Chairperson: Vladimir Kostić

11:30 - 11:45 BREAK

Short presentations

Chairperson: Miloš Stojaković

11:45 - 12:10 Mika Mattila : *Meet matrices, graphs and positive definiteness*

12:10 - 12:35 Enide A. Martins : *On the Laplacian and signless Laplacian spectrum of a graph with k pairwise co-neighbor vertices*

12:35 - 13:00 Domingos M. Cardoso : *A combinatorial simplex-like approach to graphs with convex-qp stability number using star sets*

13:00 - 15:00 LUNCH BREAK

15:00 EXCURSION

Thursday, September 19

9:00- 9:45 POSTER SESSION

9:45 - 10:00 BREAK

10:00 - 11:30
Invited lecturer: Adi Ben-Israel
Lecture 2: *Selected applications*
Chairperson: Vladimir Kostić

11:30 - 11:45 BREAK

Short presentations

Chairperson: Ana Nata

11:45 - 12:10 Célia S. Moreira : *Coupled cell networks and matrix theory*

12:10 - 12:35 Avi Berman : *Signed zero forcing*

12:35 - 13:00 Marko Orel : *Adjacency preservers*

13:00 - 15:00 LUNCH BREAK

Chairperson: Apostolos Hadjidimos

15:00 - 15:25 Isabel Giménez : *The combined matrix of a nonsingular H-matrix*

15:25 - 15:50 Maria T. Gassó : *Combined matrices of totally negative matrices*

15:50 - 16:15 Pavel A. Inchin : *Mathematical models for macro systems in the presence of limiting factors*

16:15 - 16:40 Irina N. Pankratova : *On some properties of invariant subspaces of linear operator containing cycles of rays*

16:40 - 17:00 BREAK

Chairperson: Sanja Konjik

17:00 - 17:25 Miloš Stojaković : *Small sample spaces of permutations*

17:25 - 17:50 Fatih Yilmaz : *Bidiagonal matrices and the Fibonacci and Lucas numbers*

17:50 - 18:15 Durmuş Bozkurt : *The determinant and inverse of a circulant matrix with third order linear recurrence entries*

18:15 - 18:40 Murat Gübeş : *Some well-known number sequences and their determinantal representations*

Friday, September 20

Short presentations

Chairperson: Jelena Aleksić

9:00- 9:25 Miroslav Rozložnik : *Numerical behavior of matrix splitting iteration methods*

9:25 - 9:50 Maja Nedović : *Generalizations of diagonal dominance and applications to max-norm bounds of the inverse*

9:50 - 10:15 Petr Tichý : *On matrix approximation problems that bound GMRES convergence*

10:15 - 10:40 Iwona Wróbel : *Numerical aspects of matrix inversion*

10:40 - 11:05 Krystyna Ziętak : *The dual Padé families of iterations for the matrix p-th root and the matrix p-sector function*

11:05 - 11:20 BREAK

Chairperson: Krystyna Ziętak

11:20 - 11:45 Sivakumar K.C. : *Generalized inverses and linear complementarity problems*

11:45 - 12:10 Jaroslav Horáček : *On solvability and unsolvability of overdetermined interval linear systems*

12:10 - 12:35 Milan Hladik : *Complexity issues for the symmetric interval eigenvalue problem*

12:35 - 13:00 Dragana Gardašević : *Geršgorin set for quadratic eigenvalue problem*

ABSTRACTS

Invited Speakers

Alternating sign matrices: history, patterns, completions, and spectral radius

Richard A. Brualdi

University of Wisconsin - Madison, USA

Abstract

Abstract: An alternating sign matrix (ASM) is an $n \times n$ $(0, +1, -1)$-matrix such that, ignoring 0s, in each row and column, the $+1$s and -1s alternate beginning and ending with a $+1$. We shall discuss their origins, properties, completions when only the -1s have been prescribed, and briefly the largest spectral radius possible.

Optimising the Kemeny Constant for a Markov Chain

Stephen J. Kirkland

Department of Mathematics and Statistics, University of Regina, Ireland

Abstract

Suppose that A is an irreducible stochastic matrix of order n. Then A can be thought of as the transition matrix for a Markov chain, and various quantities associated with A give insight into the nature of the corresponding Markov chain. For example, it is well–known that if A is primitive, then the stationary distribution vector w for A – that is, the left Perron vector of A normalised so that its entries sum to 1 – describes the limiting behaviour of the Markov chain, since the iterates of the latter converge to w independently of the initial distribution.

Somewhat less well–known is the Kemeny constant $K(A)$ for the Markov chain associated with A. Denoting the eigenvalues of A by $1, \lambda_2, \ldots, \lambda_n$, $K(A)$ is given by $K(A) = \sum_{j=2}^{n} \frac{1}{1-\lambda_j}$. A remarkable result of Kemeny asserts that for each $i = 1, \ldots, n, \sum_{j=1}^{n} m_{i,j} w_j = K(A) + 1$, where for $i, j = 1, \ldots, n, m_{i,j}$ denotes the mean first passage time from state i to state j. Thus, the Kemeny constant can be interpreted in terms of the expected number of time steps taken to arrive at a randomly chosen state, starting from initial state i. In particular, if $K(A)$ is small, then we can think of the Markov chain corresponding to A as possessing good mixing properties.

This last observation motivates our interest in identifying stochastic matrices for which the corresponding Kemeny constant is as small as possible. In this talk, we will give a short overview of the Kemeny constant, and discuss some results dealing with the problem of minimising the Kemeny constant over stochastic matrices that are subject various constraints. In particular, we will find the minimum value of the Kemeny constant for stochastic matrices having a specified stationary distribution vector, and characterise those stochastic matrices yielding that minimum value.

ABSTRACTS

Invited Lecturers

Lecture 1: The Moore–Penrose inverse

Adi Ben-Israel

Rutgers University, USA

Abstract

A **generalized inverse** (**G.I.**) of an arbitrary matrix $A \in \mathbb{C}^{m\times n}$ is a matrix $X \in \mathbb{C}^{n\times m}$ that satisfies certain useful properties of an inverse, and reduces to it if A is nonsingular. Several G.I.'s will be mentioned, and the most important one, the **Moore–Penrose inverse**, will be studied in detail. It is characterized as the unique solution X of the 4 Penrose equations

$$(1)\ AXA = A; \quad (2)\ XAX = X; \quad (3)\ (AX)^* = AX; \quad (4)\ (XA)^* = XA,$$

or equivalently, as the unique solution of

$$AX = P_{R(A)},\ XA = P_{R(A^*)},$$

where P_L is the orthogonal projector onto the subspace L.

(a) Existence and uniqueness.

(b) Properties.

(c) Connection to the Singular Value Decomposition.

(d) The volume of a matrix.

(e) Computations.

References

[1] R. Penrose, A generalized inverse for matrices, *Proceedings of the Cambridge Philosophical Society* **51**(1955), 406-413.

[2] A. B–I; T.N.E. Greville, *Generalized Inverses*, Springer–Verlag, 2003. ISBN 0-387-00293-6.

[3] A.B–I, A volume associated with $m \times n$ matrices, *Lin. Algeb. and its Appl.* **167**(1992), 87–111

Lecture 2: Selected applications

Abstract

The Moore–Penrose inverse of a matrix $A \in \mathbb{C}^{m\times n}$ is denoted by $A^\dagger$.

A useful property (and characterization) of $A^\dagger$ is: for any $\mathbf{b} \in \mathbb{C}^m$, the vector $x = A^\dagger\mathbf{b}$ is the **minimum** (Euclidean) **norm**, **least squares solution** (MNLSS) of the equation

$$Ax = \mathbf{b},$$

or

$$A^\dagger\mathbf{b} = \arg\min\{\|x\| : x \in \arg\min \|Ax - \mathbf{b}\|\}.$$

Most applications of $A^\dagger$ to statistics are based on this property.

An interesting application is for the orthogonal projection of an intersection of subspaces $L \cap M$

$$P_{L\cap M} = 2\,P_L\,(P_L + P_M)^\dagger\,P_M,\ \text{(Anderson \& Duffin, 1969)},$$

a closed–form alternative to the well–known asymptotic result

$$P_{L\cap M} = \lim_{n\to\infty}(P_L\,P_M)^n\,,\ \text{(Von Neumann, 1933)}.$$

Finally, applications of the matrix volume to integration and probability will be discussed.

References

[1] A. B–I; T.N.E. Greville, *Generalized Inverses*, Springer–Verlag, 2003, Chapter 8

[2] W.N. Anderson, Jr. and R.J. Duffin, Series and parallel addition of matrices, *SIAM J. Appl. Math.* **26**(1969), 576-594

[3] J. von Neumann, Functional operators vol. II. The geometry of orthogonal spaces. *Annals of Math. Studies* **22**, 1950. Princeton University Press.

[4] A. B–I, The change of variables formula using matrix volume, *SIAM Journal on Matrix Analysis* **21**(1999), 300–312

Lecture 1: Structured perturbations - normwise and componentwise

Siegfried M. Rump

Institute for Reliable Computing, Hamburg University of Technology, Germany

Abstract

Using structure in matrix algorithms sometimes leads to a dramatic improvement. For example, solving a linear system with Toeplitz matrix can be solved in $O(n^2)$ time, the same time as to print the matrix inverse. Such a solver only permits Toeplitz perturbations, thus a stability analysis should not use general perturbations. Explicit formulas are given for structured condition numbers, using normwise and componentwise perturbations. One result is that for specific right hand side the Toeplitz condition number can be exponentially better than for general perturbations. For matrix inversion things change completely. For commonly used structures the general and the structured condition number coincide. In other words, amongst the worst perturbations is a structured one. The same is true for the distance to the nearest singular matrix. It follows that the Eckart-Young Theorem is valid for structured perturbations. For componentwise perturbations things change again completely. For commonly used structures there are examples with structured condition number O(1), but arbitrarily large general condition number.

Lecture 2: Perron-Frobenius Theory for complex matrices

Abstract

It is not difficult to see that the Eckart-Young Theorem does not carry over to componentwise perturbations. One ”reason”is that the componentwise condition number of an nxn matrix can be computed in $O(n^3)$ operations, whereas computation of the componentwise distance to the nearest singular matrix is NP-hard. One may ask whether there is a relation between the reciprocal of the condition number and the distance to the nearest singular matrix for componentwise perturbations. The answer is in the affirmative, and the solution of this problem leads to a generalization of Perron-Frobenius Theory from non-negative to general real and complex matrices. A number of results nicely carry over. However, there a number of open problems. They are based on the theory but can be formulated in easy matrix terms. One example is the following. Given a real (complex) nxn matrix A such that $|A|e = ne$, where e is the vector of 1's and the absolute value is taken componentwise. Show existence of a non-trivial real (complex) vector x with $|Ax| >= |x|$. Here comparison is componentwise as well, i.e. $u >= v$ iff $u_i >= v_i$ for all i.

ABSTRACTS

Invited Speakers-Winner of YSA 2011

The role of matrix analysis in statistical genetics

Paulo Canas Rodrigues

CMA/FCT/UNL - Nova University of Lisbon, Portugal
Department of Statistics, Federal University of Bahia, Brazil

Abstract

Statistical genetics and the analysis of ”big data”are two of the major hot topics in statistics nowadays. In both cases, as for any multivariate statistical method, the use of matrix analysis is essential to understand the patterns in the data. In this talk we will present an overview about the role of matrix analysis in (plant) statistical genetics and its usefulness in dealing with high-dimensional data. Some results on weighted singular value decomposition and on how to deal with missing values in two way data tables, with genotype by environment interaction, are presented.

ABSTRACTS

Contributed Talks

Determinants of $(-1,1)$-matrices of the skew-symmetric type: a cocyclic approach

V. Alvarez, J.A. Armario, M.D. Frau and F. Gudiel

Department of Applied Mathematics and Informatics, University of Seville, Spain

Abstract

An n by n skew-symmetric type $(-1,1)$-matrix $M = [m_{i,j}]$ has 1's on the main diagonal and ± 1's elsewhere with $m_{i,j} = -m_{j,i}$. The largest possible determinant of such a matrix M is an interesting problem. The literature is extensive for $n = 0 \mod 4)$ (skew-Hadamard matrices), but for $n = 2 \mod 4)$ there are few results known for this question. In this talk we approach this problem using cocyclic matrices over the dihedral group of $2t$ elements for t odd.

Revisiting the inverse field of values problem

Natália Bebiano

Department of Mathematics, University of Coimbra, Portugal

Abstract

The field of values of a linear operator is the convex set in the complex plane comprising all Rayleigh quotients. For a given complex matrix, Uhlig proposed the inverse field of values problem: given a point inside the field of values determine a unit vector for which this point is the corresponding Rayleigh quotient. In the present note we propose an alternative method of solution to those that have appeared in the literature. Our approach builds on the fact that the field of values can be seen as a union of ellipses under a compression to the bidimensional case, in which case the problem has an exact solution. Refining an idea of Marcus and Pesce, we provide alternative algorithms to plot the field of values of a general complex matrix, that perform faster and more accurately than the known ones.

Signed zero forcing

Felix Goldberg and Avi Berman

Technion-Israel Institute of Technology, Haifa, Israel

Abstract

We introduce a new variant of zero forcing - signed zero forcing. It allows us to compute, for instance, the maximum nullity of a Z-matrix whose graph is a line graph of a clique.

The determinant and inverse of a circulant matrix with third order linear recurrence entries

Durmuş Bozkurt and Fatih Yilmaz

Department of Mathematics, Selçuk University, Konya, Turkey

Abstract

At this paper, we consider circulant matrices whose entries are third order linear recurrence elements. Then by exploiting some interesting properties of circulant matrices, we give an explicit determinant formula and some interesting properties of circulant matrices.

Jordan structure of rank one updated matrix

Rafael Bru[1], Rafael Cantó[1], Ana M. Urbano[1] and Ricardo Soto[2]

[1]Universitat Politècnica de València, Spain
[2]Universidad Católica del Norte, Antofagasta, Chile

Abstract

The relationship among eigenvalues of a given square matrix A and the rank one updated matrix $A + v_k q^T$, where v_k is a right eigenvector of A associated with the eigenvalue λ_k and q is an arbitrary vector, is well known (see Brauer's theorem [Brauer A., Limits for the characteristic roots of a matrix, Duke Math. J., 1952, 19(1), 75–91]). In this work, the relationship between the Jordan structures of A and $A + v_k q^T$ is studied. More precisely, the eigenvectors of the updated matrix in function of the eigenvectors of A are given. Further, expressions of the generalized eigenvectors (Jordan chains) of the updated matrix are given, either for left and right Jordan chains.

*This research was supported by the Spanish DGI grant MTM2010-18228.

A combinatorial simplex-like approach to graphs with convex-qp stability number using star sets

Domingos M. Cardoso[1] and Carlos J. Luz [2]

[1]Departamento de Matemática, Universidade de Aveiro, Portugal
[2]Instituto Politécnico de Setúbal, Portugal

Abstract

A graph G with convex-qp stability number is a graph for which the stability number is equal to the optimal value of a convex quadratic program P(G). There are polynomial-time procedures to recognize these graphs, except when they are adverse (that is, not complete

graphs G, without isolated vertices, such that the optimal value of P(G) and the least eigenvalue of its adjacency matrix are both integer and none of them changes when G is replaced by G' which is obtained from G deleting the neighborhood of any of its vertices). Despite several attempts, the conjecture that every adverse graph is a Q-graph is still open. In this presentation, from a recent characterization of Q-graphs based on star sets associated to the least eigenvalues of its adjacency matrix, a combinatorial simplex-like algorithm for the recognition of Q-graphs is introduced.

Orthogonal and error orthogonal models and perfect families

Aníbal Areia[1], **Francisco Carvalho**[24] **and João T. Mexia**[23]

[1]Departamento de Economia e Gestão, Instituto Politécnico de Setúbal, Portugal
[2]Centro de Matemática e Aplicações, Universidade Nova de Lisboa, Caparica, Portugal
[3]Departamento de Matemática, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, Caparica, Portugal
[4]Departamento de Matemática e Física, Instituto Politécnico de Tomar, Portugal

Abstract

We discuss, in the framework of commutative Jordan Algebras, the algebraic structure of orthogonal and error orthogonal models. We emphasize the role of perfect families of symmetric matrices. These families are basis for the commutative Jordan algebra they generate and ensure that, when normality is assumed, the models have complete sufficient statistics leading to uniformly minimum variance unbiased estimators (UMVUE) for the relevant parameters.

Keywords

Error orthogonal models, Orthogonal models, Commutative Jordan algebras, Variance components, Perfect families

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Using the fundamental matrix properties in Collocation Methods

Yücel Çenesiz and Ayşe Betül Koç

Department of Mathematics, Selçuk University, Konya, Turkey

Abstract

In this paper we investigate the usage of the operational matrices in Collocation methods such as Generalized Taylor Collocation Method (GTCM) and etc. The solution procedure is described with examples. To assess the effectiveness and preciseness of the method, handled results are compared.

Models with homoscedastic otyhogonal partition, BQUE and mixed models

João Tiago Mexia [12], Francisco Carvalho[13] and Ricardo Covas [3]

[1]Centro de Matemática e Aplicações, Universidade Nova de Lisboa, Caparica, Portugal
[2]Departamento de Matemática, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, Caparica, Portugal
[3]Departmento de Matemática e Física, Instituto Politécnico de Tomar, Portugal

Abstract

We consider models with homocedastic orthogonal partitions, such that each variance components is segregated into an orthogonal partition of $\mathbb{R}^m$. By aproaching the estimators for the variance components and the analysis of mixed models, we intend to complete the study of uniformly best linear unbiased estimators (UBLUE) within these models.

Optimal designs under the generalized extended growth curve model

Katarzyna Filipiak

Department of Mathematical and Statistical Methods, Poznań University of Life Sciences, Poland

Abstract

In this work we consider an experiment, in which several characteristics are measured in several time points. In such a case we have multi-indices matrix of observations (tensor). Assuming that the experimental units are not homogeneous, a generalized extended growth curve model is studied. Transformation of considered model using e.g. vec"operator allows to determine maximum likelihood estimators of unknown parameters by solving the set of matrix equations. To determine optimal designs we use some properties of nonnegative definite and nonnegative matrices."

Non-normal linear mixed models

Miguel Fonseca, Elsa Moreira and João Tiago Mexia

Centro de Matemática e Aplicações, FCT/UNL, Universidade Nova de Lisboa, Campus de Caparica, Portugal

Abstract

The aim of this paper is to study the efficiency of algebraic quadratic estimators for variance components. BQUE (Best Quadratic Unbiased Estimators) do not require any distribution assumptions, allowing the use of other error distributions then the ubiquitous normal distribution. Inference is studied in mixed models with non normal distributions.

Geršgorin set for quadratic eigenvalue problem

Vladimir Kostić[1] **and Dragana Gardašević** [2]

[1]Department of Mathematics and Informatics, Faculty of Science, University of Novi Sad, Serbia
[2]Belgrade Polytechnics, Belgrade, Serbia

Abstract

Quadratic eigenvalue problems appear in many applications, and the research concerning their proper treatment has drawn a lot of attention in the past few years. In some cases, however, exact computation of eigenvalues is not necessary, while their position or distribution in the complex plane is of importance. To address such situations, in this talk we will present localization techniques for quadratic eigenvalue problems that are based on the use of strictly diagonal dominant matrices. In addition, we will prove some useful properties of the obtained localization areas, and illustrate them through numerical examples.

Combined matrices of totally negative matrices

Rafa Bru, Maria T. Gassó, Isabel Giménez and Máximo Santana

Department of Applied Mathematics, Polytechnic University of Valencia, Spain

Abstract

The combined matrix of a nonsingular matrix A is the Hadamard (entry–wise) product $A \circ \left(A^{-1}\right)^T = A \circ A^{-T}$. It is well known that row (column) sums of combined matrices are constant and equal to one. For an M–matrix A, the behavior of the diagonal entries of the corresponding combined matrix was completely described in 1962 and 1964 by Fiedler. Recently, some results about combined matrices on Totally Positive matrices has been done by Fiedler and Markham in LAA-435, 2011. In this talk, we study the combined matrix of Totally Negative matrices and concentrate on the study of properties related to diagonal entries.

The combined matrix of a nonsingular H–matrix

Rafael Bru, Maria T. Gassó and Isabel Giménez

Department of Applied Mathematics, Polytechnic University of Valencia, Spain

Abstract

The combined matrix of a nonsingular matrix A, $C(A) = A \circ A^{-T}$ (Hadamard product), is related to stochastic matrices of some matrices classes and to eigenvalues of diagonalizable matrices (Horn and C.R. Johnson. Topics in Matrix Analysis, 1991):

$$B = [b_{ij}] = A \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \dots & 0 & \lambda_n \end{bmatrix} A^{-1} \quad \Rightarrow \quad C(A) \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{bmatrix} = \begin{bmatrix} b_{11} \\ b_{22} \\ \vdots \\ b_{nn} \end{bmatrix}$$

From the work of M. Fiedler (1962) one can conclude that the combined matrix of a nonsingular M–matrix is an M–matrix. In this work we extend this result for nonsingular H–matrices, that is, for all H–matrices in the Invertible class and nonsingular matrices in the Mixed class (see the H–matrices partition in Lin. Alg. Appl. 429, 2008).

Some well-known number sequences and their determinantal representations

Murat Gübeş[1] and Fatih Yilmaz [2]

[1]Department of Mathematics, Karamanoğlu Mehmetbey University, Turkey
[2]Department of Mathematics, Selçuk University, Konya, Turkey

Abstract

In this paper, we investigate some relationships between some well-known number sequences and their determinantal representations.

On Brauer-Ostrowski and Brualdi sets

Apostolos Hadjidimos

Department of Electrical & Computer Engineering, University of Thessaly, Volos, Greece

Abstract

For the localization of the spectrum of the eigenvalues of a complex square matrix, the classical Geršgorin Theorem was extended by Ostrowski who used the *generalized geometric mean* of the row and column sums of the matrix. Ostrowski, and Brauer, extended the previous idea by using generalized geometric means of products of two row and column sums. Finally, by using Graph Theory, Brualdi extended all of the previous ideas further by considering generalized geometric means of products of two or more than two row and column sums. These localization results can also provide classes of nonsingular matrices. Our main aim in this work is to exploit all the above known results and determine intervals for the parameter(s) α (α_k's) involved so that the localization of the spectrum in question as well as the determination of the associated class of nonsingular matrices are possible.

References

[1] R.A. Brualdi, *Matrices, Eigenvalues, and Directed Graphs*, Linear Multilinear Algebra 11 (1982), 143–165.
[2] Lj. Cvetković, *$H-$matrix theory vs. eigenvalue localization*, Numer. Algor. 42 (2006), 229–245.
[3] Lj. Cvetković, V. Kostić, R. Bru, F. Pedroche, *A simple generalization of Geršgorin's theorem*, Adv. Comput. Math. 35 (2011), 271–280.
[4] A. Hadjidimos, *Irreducibility and extensions of Ostrowski's Theorem*, Linear Algebra Appl. 436 (2012) 2156–2168.
[5] A. Hadjidimos, *A Note on Ostrowski's Theorem*, Linear Algebra Appl. (under revision).
[6] R.S. Varga, *Geršgorin and His Circles*, Springer-Verlag, Berlin, 2004.

Complexity issues for the symmetric interval eigenvalue problem

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Abstract

We study the problem of computing the maximal and minimal possible eigenvalue of a symmetric matrix when the matrix entries vary within compact intervals. In particular, we focus on computational complexity of determining these extremal eigenvalues with some approximation error. Besides the classical absolute and relative approximation errors, which turn out not to be suitable for this problem, we adapt a less known one related to the relative error, and also propose a novel approximation error. We show in which error factors the problem is polynomially solvable and in which factors it becomes NP-hard.

On solvability and unsolvability of overdetermined interval linear systems

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Abstract

By an overdetermined interval linear system (OILS) we mean an interval linear system with more equations than variables. By a solution set of an interval linear system $\mathbf{A}x = \mathbf{b}$ we mean

$$\Sigma = \{x | Ax = b for some A \in \mathbf{A}, b \in \mathbf{b}\}.$$

If Σ is an empty set, we call the system unsolvable.There exist many methods for computing interval enclosures of solution sets of OILS. Nevertheless, many of them return nonempty solution even if the OILS have no solution (the least squares etc.). However, in some applications we do care whether systems are solvable or unsolvable (e.g. system validation, technical computing). In our talk we would like to address the solvability and unsolvability of OILS. There is a lack of necessary and sufficient conditions for detecting solvability and unsolvability of OILS. We would like to present some newly developed

conditions and algorithms concerning these problems. The results of numerical testing will be presented and new possible ways of research stated.

Mathematical models for macro systems in the presence of limiting factors

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Abstract

Let $\mathbb{R}^n$ be n- dimensional real vector space and $F : \mathbb{R}^n \to \mathbb{R}^n$ be a map of the form $Fy = \Phi(y)Ay$ where A is $(n \times n)$- matrix and $\Phi(y)$ is a scalar function. Allocate $\mathrm{X} \subseteq \mathbb{R}^n$ such that $F : \mathrm{X} \to \mathrm{X}$. The map F generates in X the dynamical system $\{F^m, \mathrm{X}, Z_+\}$ where Z_+ is a set of nonnegative integers. In general the dynamical systems $\{F^m, \mathrm{X}, Z_+\}$ are different for different $\Phi(y)$ and A and represent a variant of generalization of one-dimensional discrete dynamical systems. Unlike the general nonlinear case the dynamical systems considered have similar properties which are determined by the matrix A and do not depend on the function $\Phi(y)$. Therefore, the systems $\{F^m, \mathrm{X}, Z_+\}$ form one class of the dynamical systems [1].

We propose this class of the systems as mathematical models with a limiting factor. Let n be a number of macro system's components, y be a vector of components' characteristics, the coefficients of the matrix A be in charge of components' interrelations and $\Phi(y)$ be a limiting function (limiting factor). Then macro system's evolution over time m is described by the system $\{F^m, \mathrm{X}, Z_+\}$. For example, the system $\{F^m, \mathrm{X}, Z_+\}$ describes the dynamics of n-group biological population with non-overlapping generations in the presence of limited resources where $y \in \mathrm{X}$ is a vector of densities of age groups, $\mathrm{X} = \{y \in \mathbb{R}^n \mid y \geq 0,\ \|y\| \leq a\}$, $a > 0$, $y = (y_1, \ldots, y_n) \geq 0$ means $y_i \geq 0$.

We develop a qualitative theory for the class of the dynamical systems $\{F^m, \mathrm{X}, Z_+\}$. The results of their analytical and numerical investigation can be used as mathematical basis for qualitative description of the dynamics of complex micro systems. This description takes into account influence of external factors, limited resources, internal interrelations and external relations with other systems and applies experimental and monitoring data implemented in the models by appropriate limiting function $\Phi(y)$ and matrix of parameters A without any limitation of number of system's components. Together with developing

methods of experimental detection of limiting factors and constructing adequate mathematical models the qualitative research of these models is an actual practical problem.

References

[1] Pankratova I.N., Cyclic Invariant Sets for One Class of Maps, Siberian Math. J., Springer, 50 (2009), no 1, 107-116.

Structured matrices in the numerical analysis of singular integral equations

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Abstract

We give some examples to show that structured matrices play an important role in both proving stability of numerical methods and designing fast algorithms for the computation of approximate solutions of Cauchy and related singular integral equations.

Generalized inverses and linear complementarity problems

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Abstract

A real square matrix A is called a Q-matrix if the linear complementarity problem $LCP(A, \mathbb{R}^n_+, q)$ has a solution for all $q \in \mathbb{R}^n$. This means that for every vector q there exists a vector x such that $x \geq 0, y = Ax + q \geq 0$ and $x^T y = 0$. A well known result of Karmardian states that if the problems $LCP(A, \mathbb{R}^n_+, 0)$ and $LCP(A, \mathbb{R}^n_+, d)$ for some $d \in \mathbb{R}^n, d > 0$ have

only zero as a solution, then A is a Q-matrix. By relaxing the condition on d and imposing a condition on the solution vector x in the two problems as above, the author introduces a new class of matrices, requiring that these two modified problems have only zero as a solution. For invertible matrices, this new class coincides with the subclass of Q-matrices. Employing the nonnegativity of certain generalized inverses, matrices belonging to this class are identified. In the process, certain well known results for Q-matrices are extended.

On convergence of SOR-like methods for solving the Sylvester equation

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Abstract

We discuss convergence of some new iterative methods for solving large-scale Sylvester matrix equation ($AX - XB = C$). The proposed algorithms belong to the class of SOR-like methods, based on the SOR (Successive Over-Relaxation) method for solving linear systems (the first of the methods was proposed by Z. Wonicki). We present two sufficient conditions under which proposed method lSOR-like is convergent. We also present an idea of changing the given matrices A and B such that C and solution X remain the same, but the convergence of any SOR-like method is improved. Some numerical experiments are given to illustrate the theoretical results and some properties of the methods.

Symplectic linear algebra over Colombeau-generalized numbers

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Abstract

We study symplectic linear algebra over the ring $\tilde{\mathbb{R}}$ of Colombeau generalized numbers. Due to the algebraic properties of $\tilde{\mathbb{R}}$ it is possible to preserve a number of central results of classical symplectic linear algebra. In particular, we construct symplectic bases for any symplectic form on a free $\tilde{\mathbb{R}}$-module of finite rank. Further, we consider the general problem of eigenvalues for matrices over $\tilde{\mathbb{K}}$ ($\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$) and derive normal forms for Hermitian and skew-symmetric matrices. Our investigations are motivated by applications in non-smooth symplectic geometry and the theory of Fourier integral operators with non-smooth symbols.
This talk is based on joint work with Günther Hörmann and Michael Kunzinger (University of Vienna).

Reverse Heinz-Kato-Furuta inequality

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[2]Department of Mathematics, University of Aveiro, Portugal
[3]Department of Physics, University of Coimbra, Portugal

Abstract

In the set up of Minkowski spaces, the Schwarz inequality holds with the reverse inequality sign. As a consequence, the same occurs with the triangle inequality. We consider extensions of this indefinite version of the Schwarz inequality. Namely, we present a reverse Heinz-Kato-Furuta inequality valid for timelike vectors and related inequalities that also hold with the reverse sign.

On some matrix aspects of design optimality in interference model

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Abstract

The aim of this paper is to present matrix formulation of finding an optimal design in various interference model. The solution of the matrix problem allows to characterize the information matrix of optimal design. This algebraic result is used to construct optimal circular block designs under interference models.

On the Laplacian and signless Laplacian spectrum of a graph with k pairwise co-neighbor vertices

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Abstract

Consider the Laplacian and signless Laplacian spectrum of a graph G of order n, with k pairwise co-neighbor vertices. We prove that the number of shared neighbors is a Laplacian and a signless Laplacian eigenvalue of G with multiplicity at least $k-1$. Additionally, considering a connected graph G_k with a vertex set defined by the k pairwise co-neighbor vertices of G, the Laplacian spectrum of G^k, obtained from G adding the edges of G_k, includes $l+\beta$ for each nonzero Laplacian eigenvalue β of G_k. The Laplacian spectrum of G overlaps the Laplacian spectrum of G^k in at least $n-k+1$ places.

Meet matrices, graphs and positive definiteness

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Abstract

Let $(P, \preceq)$ be a lattice and f be a real-valued function on P. In addition, let $S = \{x_1, \ldots, x_n\}$ be a subset of P with distinct elements. The $n \times n$ matrix having $f(x_i \wedge x_j)$ as its ij element is the *meet matrix* of the set S with respect to f and is denoted by $(S)_f$. Similarly, the *join matrix* of the set S with respect to f has $f(x_i \vee x_j)$ as its ij element and is denoted by $[S]_f$. In case when $(P, \preceq) = (Z_+, |)$ the matrices $(S)_f$ and $[S]_f$ are referred to as the GCD and LCM matrices of the set S with respect to f. Currently there are several sufficient conditions for the positive definiteness of GCD, LCM, meet and join matrices to be found in the literature (see e.g. [1,2,3]). Most of the existing results concerning positive definiteness of meet and join matrices are byproducts of certain factorizations of these matrices. In this presentation we concentrate on the positive definiteness of these matrices and use a bit different approach as earlier. In case when the set S is meet closed we give a sufficient and necessary condition for the positive definiteness of the matrix $(S)_f$. From this condition we obtain the known sufficient conditions as corollaries.The structure of any set $S \subseteq P$ can be illustrated by drawing its so called *Hasse diagram*, which can easily be interpreted as an undirected graph. It also turns out that if the graph of the set S has certain treelike structure, then the positive definiteness of the matrix $(S)_f$ depends only on the order-preserving properties of the function f.Dual theorems of these results for join matrices are presented as well. As examples we consider so called power GCD and power LCM matrices as well as MIN and MAX matrices.

References

[1] K. Bourque and S. Ligh, *Matrices associated with aritmetical functions*, Linear Multilinear Algebra 34 (1993) 261–267.[2] I. Korkee and P. Haukkanen, *On meet and join matrices associated with incidence functions*, L

Coupled cell networks and matrix theory

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Abstract

The Theory of Coupled Cell Networks was developed in the last few years by Ian Stewart, Martin Golubitsky and coworkers. In this theory, a cell is a system of ODEs and a coupled cell system is a finite collection of interacting cells. A coupled cell system can be associated with a network, a direct graph whose nodes represent cells and whose arrows represent couplings between cells. In this talk we present some important applications of Matrix Theory to the Theory of Coupled Cell Networks.

The indefinite numerical range of banded biperiodic Toeplitz operators

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Abstract

The numerical range of an operator is a well studied concept with many applications in several areas of mathematic. In this talk an infinite banded biperiodic Toeplitz matrix, regarded as an operator T acting on a Hilbert space H endowed an indefinite metric, is reduced to a family of 2×2 matrices. This approach leads to the characterization of the indefinite numerical range of this kind of operators T by performing a reduction to a 2-dimensional space taking the pseudo-convexhull of a union of hyperbolical discs. Particular attention is paid to the case of tridiagonal operators, identifying a class with indefinite hyperbolical range. These abstract results are illustrated by several examples.

Generalizations of diagonal dominance and applications to max-norm bounds of the inverse

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[2]Faculty of Technical Sciences, University of Novi Sad, Serbia

Abstract

Motivated by recent papers on max-norm bounds for the inverse of a matrix that belongs to a certain subclass of H-matrices, in this paper we presented a nonsingularity result which is a generalization of diagonal dominance property. Also, as an application, we presented new max-norm bounds for the inverse matrix and illustrated these results by numerical examples.

Adjacency preservers

Marko Orel

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Abstract

Let $\mathcal{M}$ be a set of matrices of the same size. Matrices $A, B \in \mathcal{M}$ are *adjacent* if the rank of their difference is minimal and nonzero. For many choices of $\mathcal{M}$ this means that $\text{rank}(A - B) = 1$. In that case a map $\Phi : \mathcal{M} \to \mathcal{M}$ *preserves adjacency* if $\text{rank}\big(\Phi(A) - \Phi(B)\big) = 1$ whenever $\text{rank}(A - B) = 1$. Bijections that preserves adjacency in both directions on various matrix spaces $\mathcal{M}$ were first studied by Hua in the middle of the previous century. Recently there were several developments in this area.

In this talk I will present my results that involve matrices over a finite field. Some of them and/or their proofs are closely related to other disciplines such as spectral graph theory, chromatic graph theory, geometry, special theory of relativity, etc.

On some properties of invariant subspaces of linear operator containing cycles of rays

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Abstract

Let R^n be n- dimensional real vector-space and $A \in End(\mathrm{R}^n)$ be a linear operator. A ray directed to $y \in \mathrm{R}^n$ is a set of the form $\mathrm{cone}(y) = \{\alpha y \mid \alpha \geq 0\}$, $y \neq 0$. A cycle of rays of operator A of period $p < \infty$ is determined as a system of not coinciding rays $l_1, \ldots, l_p$ for which

$$Al_1 = l_2,\ \ Al_2 = l_3, \ldots, Al_p = l_1.$$

Denote by $\ker A$ kernel of A, $\mathcal{P}(A; p, \mu) = A^p - \mu^p E$ where μ is the number, E is identity operator.

PROPOSITION 1. Let L_p be a cycle of rays of operator A of period p. Then there exist eigenvalue μ of operator A and invariant subspace $\ker \mathcal{P}(A; p, \mu)$ such that $L_p \subseteq \ker \mathcal{P}(A; p, \mu)$.

Inclusion $L_p \subseteq \ker \mathcal{P}(A; p, \mu)$ implies $\mu^p > 0$. Moreover, L_p contains in $\ker \mathcal{P}(A; mp, \mu)$ for all $m = 1, 2, \ldots$.

PROPOSITION 2. Let $\ker \mathcal{P}(A; p, \mu)$ contain a cycle of rays of operator A of period p. Then for all $y \in \ker \mathcal{P}(A; p, \mu)$ (almost all up to set of zero measure in $\ker \mathcal{P}(A; p, \mu)$) the system $(\mathrm{cone}(y), \mathrm{cone}(Ay), \ldots, \mathrm{cone}(A^{p-1}y))$ is a cycle of rays of period p.

Denote by $LCM(k, m)$ the least common multiple of integers k, m.

THEOREM. Let $\ker \mathcal{P}(A; p_j, \mu_j)$ contain a cycle of rays of operator A of period p_j, $j = \overline{1, t}$, and $|\mu_1| = \ldots = |\mu_t|$. Then

$$\sum_{1}^{t} \ker \mathcal{P}(A; p_j, \mu_j) = \ker \mathcal{P}(A; p, \mu)$$

where $p = LCM(p_1, \ldots, p_t)$ and $\mu \in \{\mu_1, \ldots, \mu_t\}$.

In this theorem for all $y \in \ker \mathcal{P}(A; p, \mu)$ (almost all up to set of zero measure in $\ker \mathcal{P}(A; p, \mu)$) the system $(\mathrm{cone}(y), \mathrm{cone}(Ay), \ldots, \mathrm{cone}(A^{p-1}y))$ is a cycle of rays of period p.

The problem of determining properties of invariant subspaces $\ker \mathcal{P}(A; p, \mu)$ arises in connection with developing a qualitative theory for a class of the dynamical systems generated by the map $Fy = \Phi(y)Ay$ where $\Phi(y)$ is a scalar function, $y \in \mathrm{X} \subseteq \mathrm{R}^n$ (see, for example [1]).

References

[1] Pankratova I.N., On Invariant Sets of a Dynamical System Generated by the Product of a Scalar Function and a Linear Vector Function, Differential Equations, MAIK Interperiodika, 45 (2009), no. 1, 142–149.

All about the $\perp$

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Abstract

For an $n \times m$ matrix A the matrix $A^{\perp}$ is defined as a matrix spanning the orthocomplement of the column space of A, when the orthogonality is defined with respect to the standard inner product $x'y$. In this talk we collect together various properties of the $\perp$ operation and its application in statistics. Results covering the more general inner products are also considered.

Inversion of Bezoutians

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Abstract

A large amount of literature is devoted to the construction of inverses of Toeplitz or Hankel matrices which are Toeplitz or Hankel Bezoutians. The converse problem - the inversion of Bezoutians - has been given short shrift up to now. Our main aim here is to fill this. We construct inverses of Bezoutians by means of solutions of corresponding Bezout equations.

Important tools are results concerning the nullspace of generalized resultant matrices. The presented formulas are also specified to cases where the Bezoutians possess additional symmetries. Inversion formulas for Hankel Bezoutians open up possibilities to construct inverses of centrosymmetric Toeplitz-plus-Hankel Bezoutians.

Supervised classifiers of ultra high-dimensional higher-order data with locally doubly exchangeable covariance structure

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[2] Royal Institute of Technology, Stockholm, Sweden

Abstract

Analysis of ultra high-dimensional higher-order data is a mathematical challenge of this Century. High-dimensional data is data with anywhere from a few dozen to thousands of dimensions, whereas ultra high-dimensional data are those that goes beyond many thousands of dimensions and can usually be obtained in the same way as high-dimensional data. However, unlike the high-dimensional data, higher-order data can be arranged in hypercubes as opposed to vectors. In this study, we develop both linear and quadratic classifiers for ultra high-dimensional third-order data, assuming that a set of different normally distributed classes with a locally doubly exchangeable covariance structure [1] and with a constant mean vector over space is given.

We derive a two-stage procedure for estimating the covariance matrix: at the first stage, the Lasso-based structure learning is applied to sparsifying the block components within the covariance matrix. At the second stage, the maximum likelihood estimators of all block-wise parameters are derived given that the within block covariance structure is doubly exchangeable and the mean vector has a Kronecker product structure. We also study the effect of the block size on the classification performance in the ultra high-dimensional setting and derive a class of asymptotically equivalent block structure approximations [2], in a sense that the choice of the block size is asymptotically negligible.

We explore the performance accuracy of our new supervised decision rules for ultra high-dimensional higher-order data and show that these decision rules are very efficient in learning by very small sized training samples and then successfully classifying the test samples.

Keywords

Classification rule, Class of asymptotically equivalent structure approximations, Locally doubly exchangeable covariance structure, Graphical Lasso, Maximum likelihood estimates, Ultra high-dimensional higher-order data.

References

[1] Leiva, R. and Roy, A. (2012). Linear discrimination for three-level multivariate data with separable additive mean vector and doubly exchangeable covariance structure. *Computational Statistics and Data Analysis 56(6)*, 1644–1661.
[2] Pavlenko, T., Björkstrom A. and Tillander, A. (2012). Covariance Structure Approximation via gLasso in High-Dimensional Supervised Classification. *Journal of Applied Statistics* 1–24.

Numerical behavior of matrix splitting iteration methods

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Abstract

In this contribution we study numerical behavior of stationary and two-step splitting iteration methods for solving large sparse systems of linear equations. We show that inexact solutions of inner linearsystems associated with the matrix splittings may considerably influence the convergence and the accuracy of the approximate solutions computed in finite precision arithmetic. We analyze several mathematically equivalent implementations and find the corresponding componentwise or normwise forward or backward stable implementations.

Small sample spaces of permutations

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Abstract

We start by introducing the notion of a small sample space (SSS), with few illustrative examples and applications. Then, we take a closer look at SSS of permutations with respect to the so-called min-wise independence. En route, we will use notions and techniques from theoretical computer science, linear algebra, combinatorics, probability theory - all on fairly basic level. Min-wise independent SSS are essential for algorithms for detecting near-duplicate documents, some of which are used in practice by the Web indexing software. Also, they are used for derandomization of randomized algorithms that require suitable random input.

Estimation of extremal singular values

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Abstract

In estimating the largest and the smallest singular values in the class of matrices equiradial with a given n-by-n complex matrix A it was proved in [3] that they are attained at one of $n(n-1)$ sparse nonnegative matrices and at one of $n(n-1)$ sparse M-matrices when there is one, respectively. Next, in [2] some circumstances were identified under which the set of possible optimizers of the largest singular value can be further narrowed. Herewe show that in the case $n = 3$ the extremal singular values are attained at one of 3 sparse nonnegative matricesand at one of 3 sparse M-matrices. The possible subset of three depends in a simple way on the data. Moreover, we establish a matrix that maximizes the

largest singular value and a matrix that minimizes the smallest singular value in the class of all complex 3-by-3 upper triangular matrices equimodular with any of six matrices related, via permutations of off-diagonal entries, to a given complex 3-by-3 upper triangular matrix. Finally, as a bye-product of these considerations, an inequality between the spectral radius of a 3-by-3 nonnegative matrix X and the spectral radius of a modification of X is also proposed.

Diagonal dominance in Euclidian norm

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Abstract

It is well known that non-singularity of one special class of matrices, named SDD matrices, is at its core analogous with the principles established in the classical Geršgorin's theorem. Moreover, the concept of SDD matrix non-singularity is firmly based on the grounds of maximum matrix norm. Although direct switch to other norm may not result in significant overall benefits right from the start, Euclidean norm approach led to some compelling outcomes. The focus of our work is to induce this concept to some other class of H-matrices, especially to those of S-SDD and alpha type, and discuss plausible benefits concerning problems in various fields of applied linear algebra.

Affine Hadamard families stemming from Kronecker products of Fourier matrices

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Abstract

A complex Hadamard matrix (CHM) is a matrix with unimodular entries and orthogonal rows and columns. An affine Hadamard family (AHF) stemming from a CHM H has the form $\{H \circ \mathbf{EXP}(\mathbf{i}R) : \ R \in \mathcal{R}\}$ and consists purely of CHM's(where $\circ$ is the entrywise product, $\mathcal{R}$ is a linear subspace of $\mathbb{R}^{N\times N}$, $\mathbf{EXP}$ is the entrywise exponentiation) . Consider a cyclic Fourier matrix $[F_N]_{i,j} = e^{\mathbf{i}\frac{2\pi}{N}ij}$, indexed by $\mathcal{I}_{F_N} = \mathbb{Z}_N \ni i,j$. More generally consider an abelian Fourier matrix
$F = F_{N_1} \otimes \ldots \otimes F_{N_r}$, $[F]_{(i_1,\ldots,i_r),(j_1,\ldots,j_r)} = [F_{N_1}]_{i_1,j_1} \cdot \ldots \cdot [F_{N_r}]_{i_r,j_r}$,
indexed by $\mathcal{I}_F = \mathbb{Z}_{N_1} \times \ldots \times \mathbb{Z}_{N_r} \ni (i_1,\ldots,i_r),(j_1,\ldots,j_r)$. We provide a construction of a wide class of AHF's stemming from F and conjecture they cannot be extended. Our construction is based on decompositions of the indexing group $\mathcal{I}_F$. It generalizes our previous methodworked out for cyclic Fourier matrices in a group theoretic language.

Keywords

Complex Hadamard matrix, Fourier matrix, Kronecker product.

On matrix approximation problems that bound GMRES convergence

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[2]Institut für Mathematik, Technische Universität Berlin, Berlin, Germany
[3]Institute of Computer Science, Academy of Sciences of the Czech Republic, Prague, Czech Republic

Abstract

In this talk we study two matrix approximation problems that provide a bound on the GMRES convergence, the first one called *worst-case GMRES* and the second one *ideal GMRES*. We summarize known results and present new results on their characterization and uniqueness of the solution.

Extremal values of modified equiradial and equimodular sets of a given matrix

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Abstract

Equiradial and equimodular sets of a given matrix, as well as their modifications, are constantly in range of interest of many mathematicians. We present some results related to matrices from the mentioned sets. We characterize, among others, the best possible estimates of a matrix with the largest spectral radius within the set, and the smallest (in terms of the absolute value) eiganvalue.

Numerical aspects of matrix inversion

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Abstract

Matrix inversion has numerous applications in statistics, cryptography, computer graphics, etc. It is hard to imagine a computation system or a programming environment without a

library or a function that calculates the inverse of a matrix. However, not every software uses algorithms that give accurate results. Moreover, there is a variety of new papers dealing with numerical algorithms for matrix inversion, whose authors neglect the issue of numerical stability of their algorithms and focus only on complexity (number of arithmetic operations). However, not all proposed algorithms have good numerical properties.We present a comparison of certain algorithms for computing the inverse of a given matrix. We will focus mainly on algorithms for structured (mostly banded) matrices. We study the numerical properties of considered algorithms. Numerical experiments in MATLAB are given to compare the performance and accuracy of some methods for computing the matrix inverse.

Factorization of Pascal matrices via bidiagonal matrices

Fatih Yilmaz and Durmuş Bozkurt

Selçuk Üniversitesi, Fen Fakültesi, Matematik Bölümü, Konya, Turkey

Abstract

At this study, we consider Pascal matrices and then, we get some factorization formulas using some interesting properties of some well-known number sequences.

The dual Padé families of iterations for the matrix pth root and the matrix p-sector function

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Abstract

In the talk we focus on the Padé family of iterations , introduced by Laszkiewicz and Ziętak in 2009, for computing the principal matrix pth root and the new dual Padé families for computing the principal matrix pth root and the matrix p-sector function. We determine certain regions of convergence of these iterations.We show that some properties of the Newton and Halley methods, presented by Guo in 2010, follow from properties of the Padé approximants that generate the iterations of the Padé and dual Padé families. We also investigate properties of the iterates, generated by iterations of the dual Padé family for computing the matrix pth root.

ABSTRACTS

Contributed Posters

SDD property for matrix operators on l^p spaces and its application

Jelena Aleksić, Vladimir Kostić and Milica Žigić

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Abstract

We investigate the resolvent set of an infinite matrix that can act as a linear operator from the Banach space l^p to itself. To that end, we propose a generalization of the standard strict diagonal dominance that is adapted to l^p and l^q norms, $\frac{1}{p} + \frac{1}{q} = 1$, and obtain new localization of the resolvent sets.

Generally balanced nested row-column designs for near-factorial experiments

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Abstract

In this study nested row-column designs (NRC) adequate for near-factorial experiments are considered. This kind of experiments is defined as the experiment in which treatments being the combinations of two experimental factors A and B occur both with the control treatment, which is not the combination of levels of this factors (hence $v = ab + 1$). The analysis of NRC designs is based on the mixed model [1, 2] and the method of analysis of the so called multistratal experiments of the orthogonal block structure proposed by Nelder [3, 4] is used. The analysis is related to four strata: between blocks (1), between rows (2), between columns (3) and the bottom stratum (4), the so called "rows-by-columns stratum". Moreover, all considered designs have the general balance property, thus in every strata we can consider the estimation of the same set of basic contrasts. In this study some new classes of NRC designs are defined. The general formulas of information matrices, its generalized inverses and the forms of efficiency factors for designs from the considered class are given.

References

[1] Bailey, R.A., Williams E.R., 2007. Optimal nested row-column designs with specified components. Biometrika 94: 459–468.
[2] Łacka, A., Kozłowska, M., Bogacka, B., 2009. Estimation and testing hypotheses in a block design with nested rows and columns. Biometrical Letters 46: 113–128.
[3] Nelder, J.A., 1965a. The analysis of randomized experiments with orthogonal block structure. I. Block structure and the null analysis of variance. Proc. Roy. Soc. London 283 Ser. A: 147–162.
[4] Nelder, J.A., 1965b. The analysis of randomized experiments with orthogonal block structure. II. Treatment structure and the general analysis of variance. Proc. Roy. Soc. London 283 Ser. A: 163–178.

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