Lecture 2: Selected applications

The Moore–Penrose inverse of a matrix $A \in \mathbb{C}^{m\times n}$ is denoted by $A^\dagger$.

A useful property (and characterization) of $A^\dagger$ is: for any $\mathbf{b} \in \mathbb{C}^m$, the vector $\mathbf{x} = A^\dagger\mathbf{b}$ is the **minimum** (Euclidean) **norm**, **least squares solution** (MNLSS) of the equation

$$A\mathbf{x} = \mathbf{b},$$

or

$$A^\dagger\mathbf{b} = \arg\min\{\|\mathbf{x}\| : \mathbf{x} \in \arg\min \|A\mathbf{x} - \mathbf{b}\|\}.$$

Most applications of $A^\dagger$ to statistics are based on this property.

An interesting application is for the orthogonal projection of an intersection of subspaces $L \cap M$

$$P_{L\cap M} = 2\,P_L\,\left(P_L + P_M\right)^\dagger\,P_M,\ \text{(Anderson \& Duffin, 1969)},$$

a closed–form alternative to the well–known asymptotic result

$$P_{L\cap M} = \lim_{n\to\infty}\left(P_L\,P_M\right)^n,\ \text{(Von Neumann, 1933)}.$$

Finally, applications of the matrix volume to integration and probability will be discussed.

References

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