

Lecture 2: Selected applications

The Moore–Penrose inverse of a matrix $A \in \mathbb{C}^{m \times n}$ is denoted by A^\dagger .

A useful property (and characterization) of A^\dagger is: for any $\mathbf{b} \in \mathbb{C}^m$, the vector $\mathbf{x} = A^\dagger \mathbf{b}$ is the **minimum** (Euclidean) **norm, least squares solution** (MNLSS) of the equation

$$A\mathbf{x} = \mathbf{b},$$

or

$$A^\dagger \mathbf{b} = \arg \min \{ \|\mathbf{x}\| : \mathbf{x} \in \arg \min \|A\mathbf{x} - \mathbf{b}\| \}.$$

Most applications of A^\dagger to statistics are based on this property.

An interesting application is for the orthogonal projection of an intersection of subspaces $L \cap M$

$$P_{L \cap M} = 2 P_L (P_L + P_M)^\dagger P_M, \text{ (Anderson \& Duffin, 1969),}$$

a closed–form alternative to the well–known asymptotic result

$$P_{L \cap M} = \lim_{n \rightarrow \infty} (P_L P_M)^n, \text{ (Von Neumann, 1933).}$$

Finally, applications of the matrix volume to integration and probability will be discussed.

References

- [1] A. B–I; T.N.E. Greville, *Generalized Inverses*, Springer–Verlag, 2003, Chapter 8
- [2] W.N. Anderson, Jr. and R.J. Duffin, Series and parallel addition of matrices, *SIAM J. Appl. Math.* **26**(1969), 576–594
- [3] J. von Neumann, Functional operators vol. II. The geometry of orthogonal spaces. *Annals of Math. Studies* **22**, 1950. Princeton University Press.
- [4] A. B–I, The change of variables formula using matrix volume, *SIAM Journal on Matrix Analysis* **21**(1999), 300–312